A Bayesian framework for optimal motion planning with uncertainty

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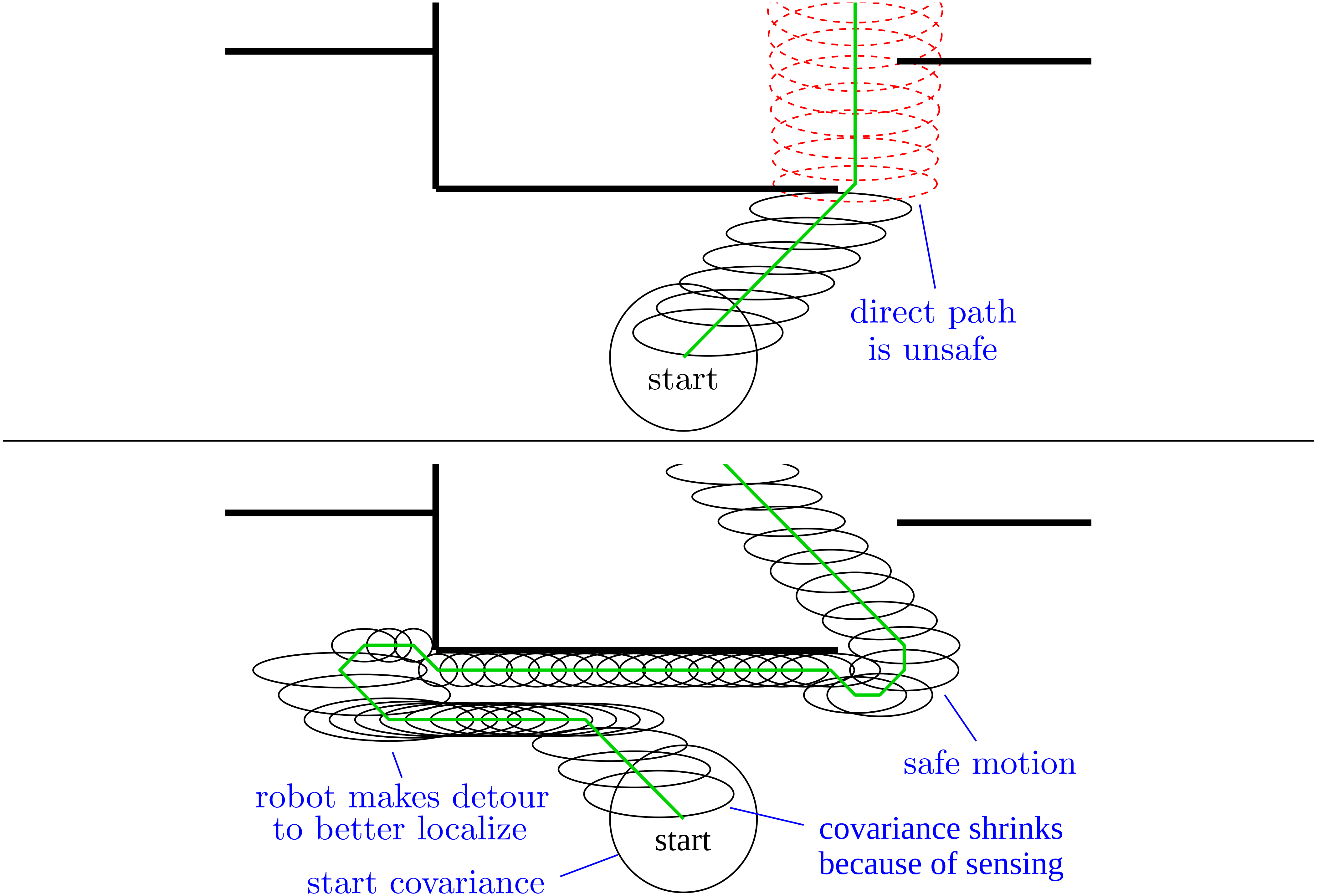


direct path
is unsafe
start
safe motion
robot makes detour
to better localize
start
covariance shrinks
because of sensing
start covariance

Many formalizations, many approaches

- **Preimage-back chaining**: Lozano-Perez *et al.* (1984); Lazanas and Latombe (1992); Fraichard and Mermond (1998)
- **Sensor-based planning**: Bouilly *et al.* (1995); Khatib *et al.* (1997)
- **The Information Space approach**: Barraquand and Ferbach (1995); O'Kane and LaValle (2005); O'Kane (2006); O'Kane and LaValle (2006)
- **Sensor uncertainty fields (SUF):** Takeda and Latombe (1992); Takeda *et al.* (1994); Trahanias and Komninos (1996); Vlassis and Tsanakas (1998); Makarenko *et al.* (2002)
- **Set-membership approach**: Page and Sanderson (1995b,a)
- **Dynamic programming**: Blackmore *et al.* (2006); Blackmore (2006)
- $A^\star$**, RRT**: Lambert and Fort-Piat (2000); Lambert and Gruyer (2003); Gonzalez and Stentz (2007)

Dimensions

- **How to represent uncertainty?**
  - Uncertainty is a bounded set.
  - Probabilistic ([isotropic] covariances, compressed information space, ...)
- **How does the uncertainty accumulate?** Bayesian, linearly with distance, ...
- **How does the uncertainty shrink?** Bayesian, "reset" to zero...
- **Which problem to solve?**
  - find a safe path, minimizing the execution time
  - find a safe path, minimizing the final covariance
  - maximize the collected information, with free final pose, ...
- **How to represent the plan/policy?**

Our approach – overview

- We work in the space poses$\times$covariances.
  - Already used in Lambert and Gruyer (2003)
  - We are more careful with assumptions.
  - We define transitions independently of localization algorithm.
- We consider two problems: minimize final time and final covariance.
- We develop two algorithms:
  - *forward*: $A^\star$-like with propagation of states
  - *backward*: backprojection of constraints from target to goal
- Emphasis on **exploiting problem structure** with generic search framework based on **dominance relations**.

Motion planning with uncertainty

Find a continuous function $\boldsymbol{q}^\star(\mathrm{t})$ such that:

$$\boldsymbol{q}^\star(0) = \boldsymbol{q}_{\text{start}} \qquad \boldsymbol{q}(0) \sim p_0(\boldsymbol{q})$$

$$\boldsymbol{q}^\star(t) \in \mathcal{C}_{\text{free}} \qquad \mathbb{P}(\boldsymbol{q}(t) \in \mathcal{C}_{\text{free}}) \geq 1 - \epsilon$$

$$\boldsymbol{q}^\star(t_f) \in \mathcal{C}_{\text{target}} \qquad \mathbb{P}(\boldsymbol{q}(t_f) \in \mathcal{C}_{\text{target}}) \geq 1 - \epsilon$$

(+ kinematic/dynamic constraints) (+ model for robot/sensors)

$$\min J(\boldsymbol{q}^\star, t_f) \qquad \min E\{J(\boldsymbol{q}^\star, t_f)\}$$

- In general, the solution is a function from the space of probability distribution of the state to the space of actions.

Approach: PP with uncertainty $\simeq$ PP in the pose$\times$covariance space

- We reduce the problem to deterministic planning in the space $\mathcal{S}$ = pose$\times$covariance:

$$\boldsymbol{q}(0) \sim p_0(\boldsymbol{q}) \qquad\qquad \boldsymbol{s}_0 = \langle \boldsymbol{q}_0, \boldsymbol{\Sigma}_0 \rangle$$

$$\mathbb{P}(\boldsymbol{q}(t) \in \mathcal{C}_{\text{free}}) \geq 1 - \epsilon \qquad\qquad \boldsymbol{s}_t \in \mathcal{S}_{\text{free}}$$

$$\mathbb{P}(\boldsymbol{q}(t_f) \in \mathcal{C}_{\text{target}}) \geq 1 - \epsilon \qquad\qquad \boldsymbol{s}_{t_f} \in \mathcal{S}_{\text{target}}$$

- $\mathcal{S}_{\text{free}}$ e $\mathcal{S}_{\text{target}}$ are defined using bounds on the covariances:

$$\boldsymbol{s}_t \in \mathcal{S}_{\text{free}} \quad \Leftrightarrow \quad \boldsymbol{q}_t \in \mathcal{C}_{\text{free}} \quad \wedge \quad \boldsymbol{\Sigma}_t \leq \textsc{constraints}(\boldsymbol{q}_t)$$

$$\boldsymbol{s}_t \in \mathcal{S}_{\text{target}} \quad \Leftrightarrow \quad \boldsymbol{q}_t \in \mathcal{C}_{\text{target}} \quad \wedge \quad \boldsymbol{\Sigma}_t \leq \mathbf{M}$$

- The set $\textsc{constraints}(\boldsymbol{q}_t)$ depends on the geometry of the environment.

Evolution of uncertainty

- Let $\boldsymbol{\Sigma}_u$ be the odometry error, and $\boldsymbol{\mathcal{I}}(\boldsymbol{q})$ the Fisher information matrix. Then for the covariance of the estimate:

$$\boldsymbol{\Sigma}_k \quad \star \quad \left(\boldsymbol{\mathcal{I}}(\boldsymbol{q}_k) + (\boldsymbol{\Sigma}_{k-1} + \boldsymbol{\Sigma}_u)^{-1}\right)^{-1}$$

(note: simplified formula) where $\star$ is:

  - "$=$" in the linear case.
  - "$\geq$" is the Bayesian Cramér-Rao bound for unbiased estimators.
  - "$\simeq$" in practice, at least for range-finders (see ICRA'07 paper)

- Semi-formal assumptions:
  - The distribution is $\simeq$ Gaussian during the optimal motion.
  - The localization algorithm is unbiased and $\simeq$ efficient.
  - The uncertainty of the pose is small with respect to the complexity of the environment: $\boldsymbol{\mathcal{I}}(\boldsymbol{q}) \simeq \boldsymbol{\mathcal{I}}(\hat{\boldsymbol{q}})$.

Problems considered

We study two problems:

- Minimizing the final time.
- Minimizing the final covariance (with a bound on the time).

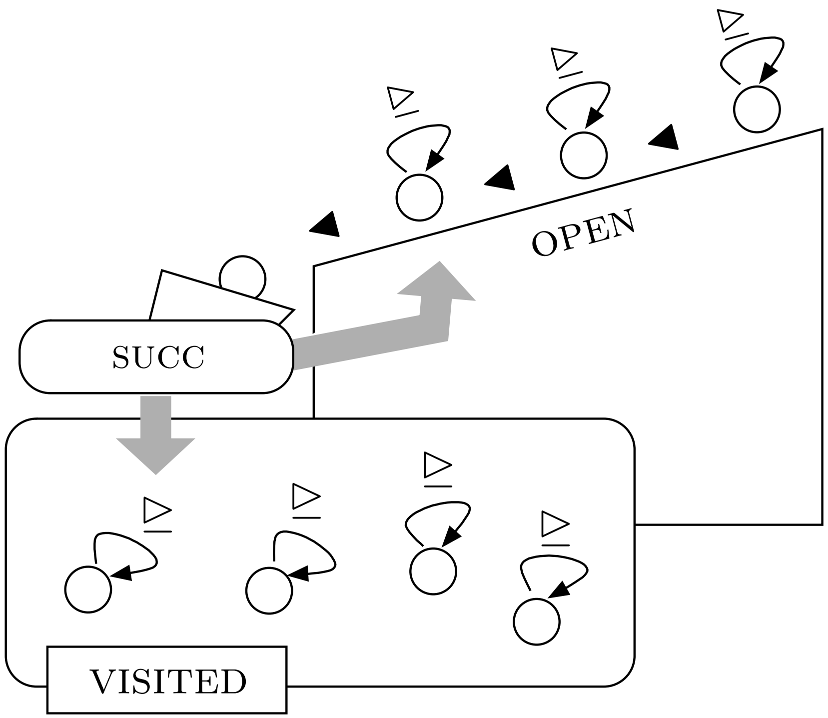
$$\min_{\leq} \mathbf{\Sigma}(t_f) \qquad \text{subject to } t_f \leq t_{\max}$$

Lots of differences with respect to standard motion planning:

- There is, in general, a continuity of solutions.
- Solutions are not reversible.
- Because sensors have specific frequency, time is important, not merely a parameterization.
- Much of the complexity comes from the fact that $\leq$ is not a total order for covariances.

Planning by searching

- The generic search algorithm has two relations:
  - a partial order $\trianglerighteq$ used for dominance (discarding nodes)
  - a total order $\blacktriangleleft$ used for precedence (search direction)



Put n_0 in OPEN.
while OPEN is not empty **do**
    Pop first (according to $\blacktriangleleft$) node $\boldsymbol{n}$ from OPEN.
    for all $\boldsymbol{s}$ in $SUCCESSORS(\boldsymbol{n})$ **do**
        Report success if $IS\_GOAL(\boldsymbol{s})$.
        Ignore $\boldsymbol{s}$ if it is $\trianglerighteq$-dominated in VISITED.
        Discard nodes in VISITED $\trianglerighteq$-dominated by $\boldsymbol{s}$.
        Put $\boldsymbol{s}$ in VISITED.
        Discard nodes in OPEN $\trianglerighteq$-dominated by $\boldsymbol{s}$.
        Put $\boldsymbol{s}$ in OPEN.
    end for
end while
Report failure.

Forward approach

- Nodes are tuples $\boldsymbol{n} = \langle \boldsymbol{q}, \boldsymbol{\Sigma}, t \rangle$:

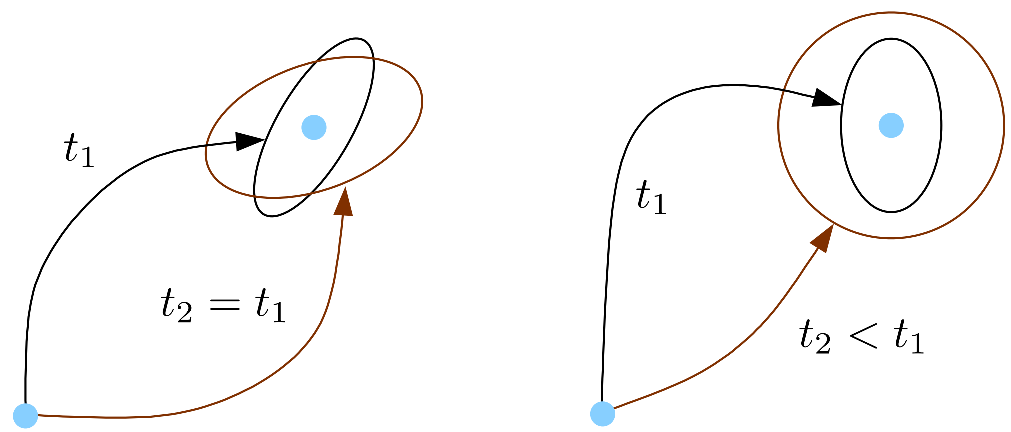
  "Can go from $\boldsymbol{q}_{start}$ to $\boldsymbol{q}$ in time t with final covariance $\boldsymbol{\Sigma}$."

- Search starts from <u>initial</u> pose: $\boldsymbol{n}_0 = \left\langle \boldsymbol{q}_{\text{goal}}, \boldsymbol{\Sigma}_0, 0 \right\rangle$.

- Most of the work is the definition of dominance relations $(\boldsymbol{n}_1 \trianglerighteq \boldsymbol{n}_2)$ for discarding nodes. Basic example (there are more powerful ones):

$$(\boldsymbol{n}_1 \trianglerighteq \boldsymbol{n}_2) \Leftrightarrow (\boldsymbol{q}_1 = \boldsymbol{q}_2) \wedge (t_1 \leq t_2) \wedge (\boldsymbol{\Sigma}_1 \leq \boldsymbol{\Sigma}_2)$$

- Example of two nodes that are <u>not</u> comparable:

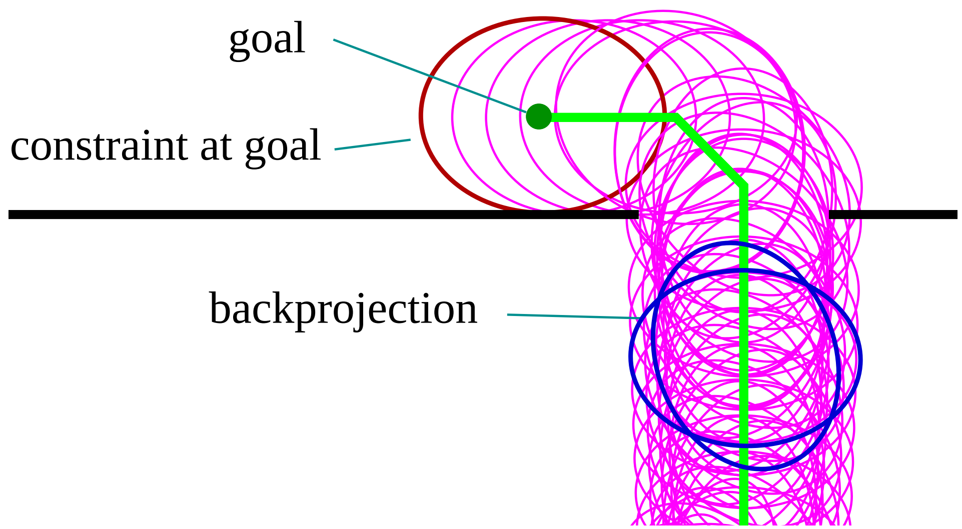


Backward approach

- Nodes are tuples $\boldsymbol{n} = \langle \boldsymbol{q}_k, \{\mathbf{M}_i\}, tg \rangle$:

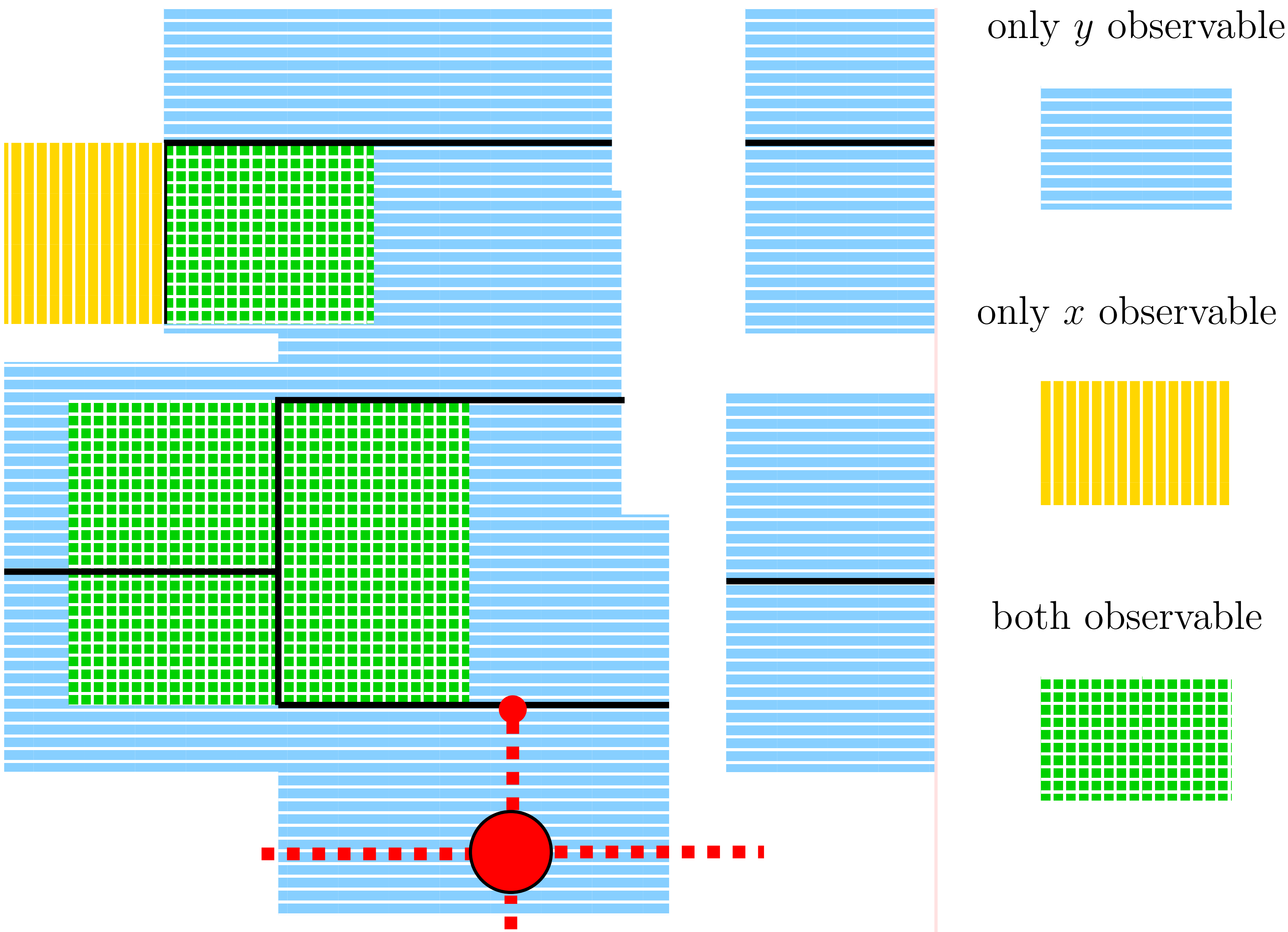
"If in $\boldsymbol{q}_k$, and $\boldsymbol{\Sigma}_k \leq \{\mathbf{M}_1, \mathbf{M}_2, \ldots\}$, then I can arrive to $\boldsymbol{q}_{goal}$ in time tg."

- Search starts from <u>final</u> pose: $\boldsymbol{n}_0 = \left\langle \boldsymbol{q}_{\text{goal}}, \textsc{constraints}(\boldsymbol{q}_{\text{goal}}), 0 \right\rangle$.
- Constraints are back-propagated.

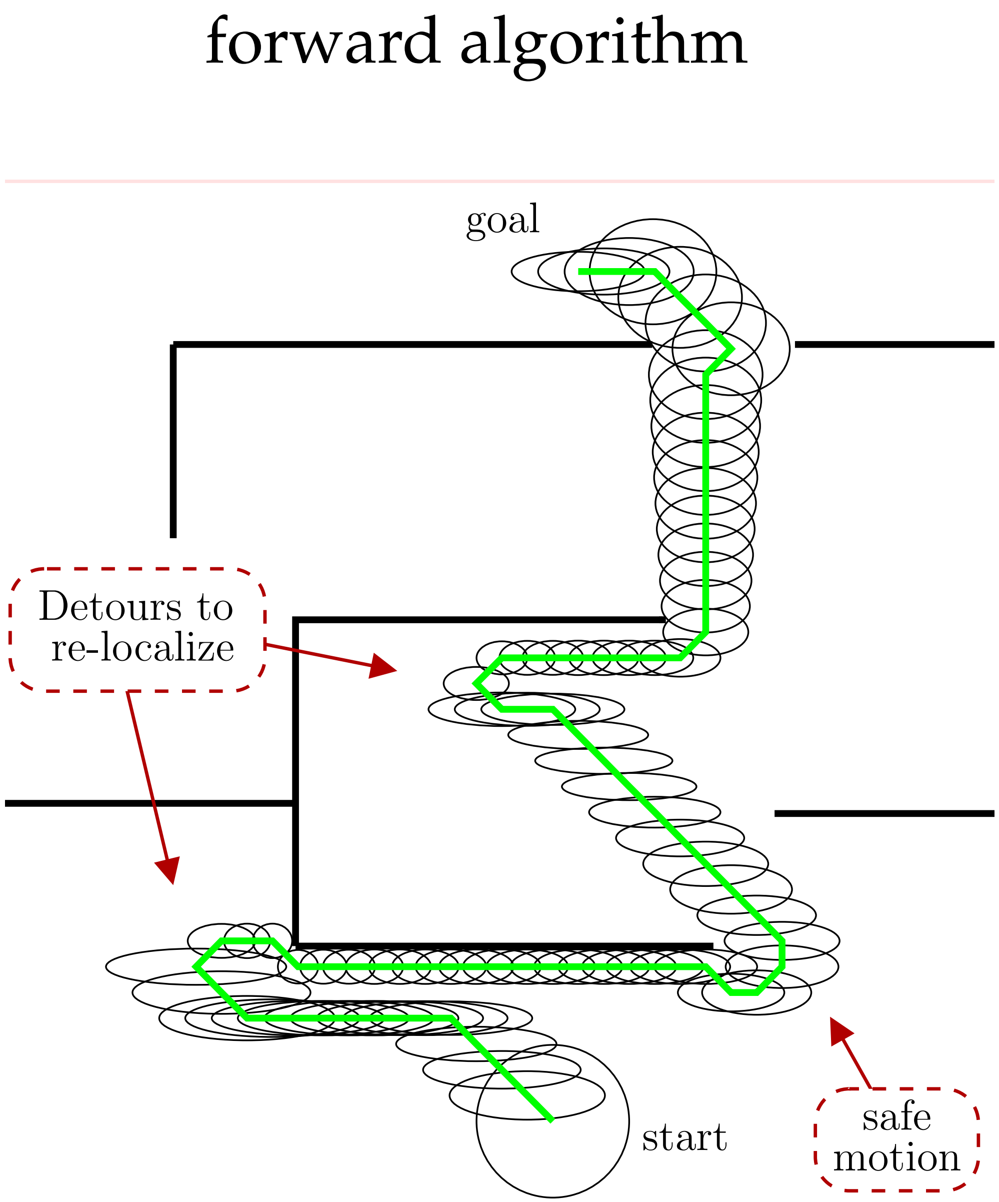


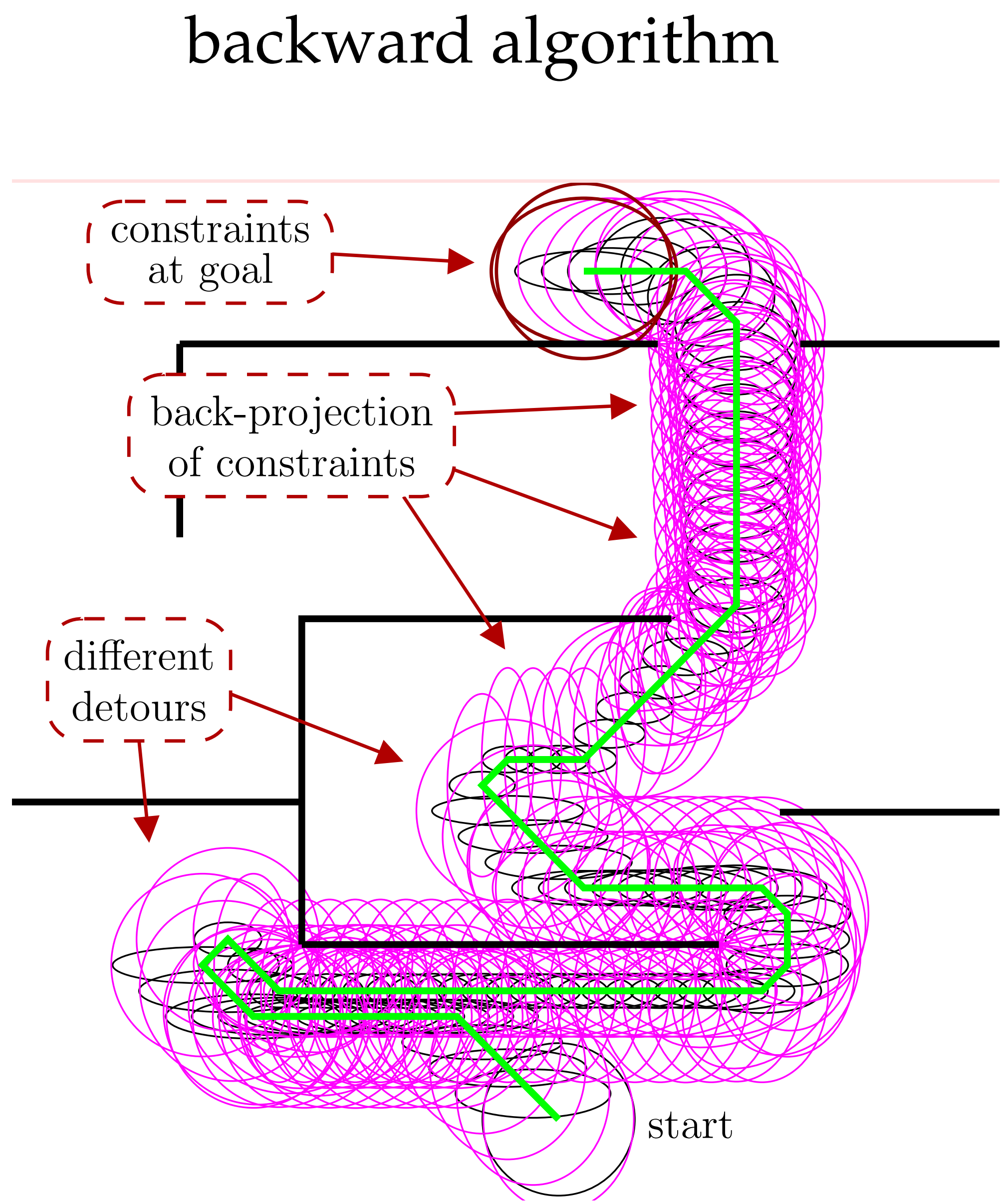
- Dominance relations are really ugly to show.

Example: Fisher information matrix

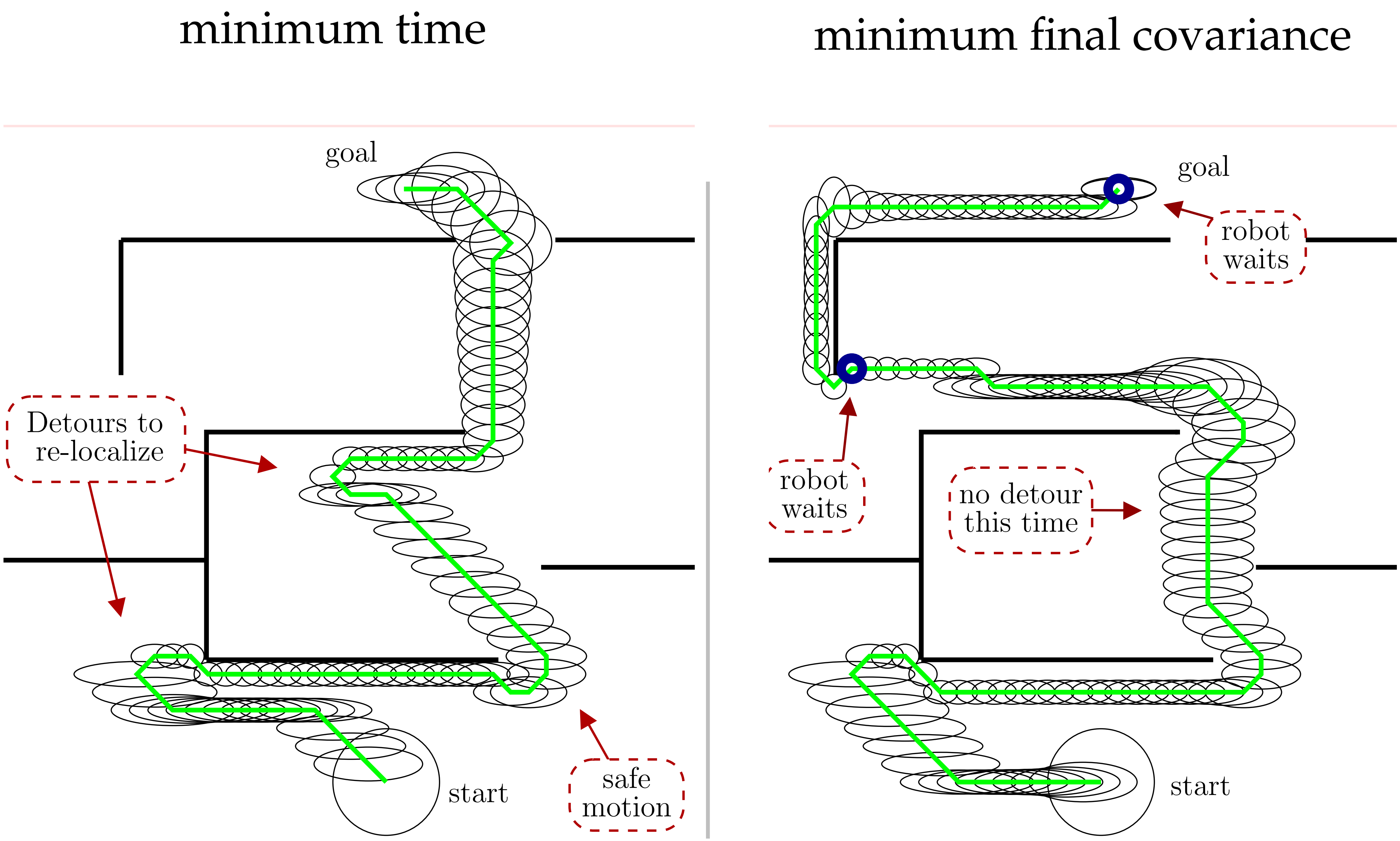


Example: minimum time path





Minimum time vs. minimum final covariance



Conclusions (things we learned)

- On planning with uncertainty:
  - Nice formalization using Fisher information matrix.
  - Nice problem(s), with many peculiar properties.
  - Lots of structure that can be exploited.
- On solving through the two algorithms:
  - Dominance relations are great for analysis and implementation.
  - Two optimal algorithms giving different solutions.
- Bad ideas met along the way:
  - Do not discretize covariances! (lose correctness)
  - Representation matter (inverse of covariance might be better)
- Source code and cool animations available at my website.

Computational cost

$A^{\star}$:	forward	backward	no uncertainty
created nodes	5'474	10'110	369
expanded nodes	4'211	7'321	229
nodes still active at the end	4'477	7'337	328
matrix comparisons	106'252	1'021'156	-
time - G4 1.5GHz	0.51	2.13	0.04
time - P4 2.8GHz	0.23	1.06	0.02

- Backward tree can be re-used if goal remains the same.

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