HSM3D:
Feature-Less Global 6DOF Scan-Matching in the Hough/Radon Domain

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3D data alignment how-to

- The data is 3D, but the *sensor motion* is 2D?
  (e.g., 3D scanner on a car)
  - ⇒ Then adapt known 2D methods to work on 2D slices of data.
  - see Olson, "Real-Time Correlative Scan Matching" SaD10.2
- Do you have a good initial guess?
  - ⇒ Then use an iterative registration procedure.
  - see next talk, "... a comparison of ICP and NDT"
- Are there consistent, regular, features? (e.g., planar surfaces)
  - ⇒ Then use some feature matching scheme (RANSAC, etc.)
- No features, unstructured data, no initial guess, general 3D motion?
  - keep listening...

“Local” and “global” algorithms

- Challenges of **local** algorithms:
  - How to establish correspondences?
  - How to speed up convergence?
  - How to avoid local minima? (re-weighting, outliers, etc.)
- Challenges of **global** 6DOF algorithms:
  - Must deal with multiple solutions.
  - Must reduce the 6DOF problem $\Rightarrow$ use of invariants
  - If using voting procedures, trade-off with cell size:
    - bigger: more robust, faster
    - smaller: more precise
  - “Fancy” math is sometimes necessary (no Euler angles...)

Related work

- Makadia et al, “Fully automatic registration of 3D point clouds” Computer Vision and Pattern Recognition, 2006.
    - Creates a *translation-invariant* statistics
        - histogram of surfaces having the same orientation
    - This decouples rotation and translation
        - same general idea of HSM3D
    - **Cons**: needs smooth surfaces, fails in simple cases (spheres).
- Reyes, Medioni, Bayro, “Registration of 3D points using geometric algebra and tensor voting,” Int. J. Computer Vision, 2007.
    - **Cons:** *really* hard to understand
    - **Cons:** complexity quadratic in the number of points
- <u>Huge</u> literature of (tangentially) related problems (registration of volumes, etc.).

HSM3D

3D version of the algorithm presented in: *Censi, Grisetti, Iocchi, "Scan matching in the Hough domain", ICRA 2005.*

- **Design goal:** fast global estimate(s) with limited precision.
- **Input:** two sets of (oriented) 3D points.
- **Output:** ordered set of candidate roto-translations.
- **Overview:**

  1. Compute the 3D Hough (Radon) transform.
     - Uses whole Hough transform, not just the peaks (no planes necessary!)
  2. Compute the two Hough Spectra.
     - This will be our translation-invariant.
  3. Compute rotations hypotheses by matching the two spectra.
  4. Given the rotation, find the translation
  5. Rank the hypotheses.

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Hough/Radon transform

Definition. The Hough Transform maps a 3D image into a function defined on $\mathbb{S}^2 \times \mathbb{R}$.

$$\mathrm{HT} : (\mathbb{R}^3 \to \mathbb{R}^+) \to \left(\mathbb{S}^2 \times \mathbb{R} \to \mathbb{R}^+\right)$$

Given a kernel $k : \mathbb{R} \to \mathbb{R}^+$, the value of the HT of i at point $(s, \rho) \in \mathbb{S}^2 \times \mathbb{R}$ is defined as:

$$\mathrm{HT}[i](s, \rho) = \int_{\mathbb{R}^3} i(v) k \left(\langle s, v \rangle - \rho\right) \, dv \tag{1}$$

- $\mathbb{R}^3 \to \mathbb{R}^+$ is, generalizing, the point cloud.
- $\mathbb{S}^2 \times \mathbb{R}$ is the sets of planes (direction $s \in \mathbb{S}^2$, distance from origin $\rho \in \mathbb{R}$)
- It is not necessary for the points to contain planes for the HT to be meaningful.
- We also define an *oriented* HT transform if points come with a normal.

Hough Spectrum

The Hough Spectrum (HS) maps a 3D "image" to a function defined on the sphere:

$$\mathrm{HS} : (\mathbb{R}^3 \to \mathbb{R}^+) \to \left(\mathbb{S}^2 \to \mathbb{R}^+\right)$$

$$\mathrm{HS}[i](s) \quad = \quad ||\mathrm{HT}[i](s, \cdot)||_2$$

Properties:

- the HS is **invariant to translations** of the input:

$$\mathrm{HS}[i \circ t] = \mathrm{HS}[i]$$

- the HS is **rotated by a rotation** of the input:

$$\mathrm{HS}[i \circ r] = \mathrm{HS}[i] \circ r$$

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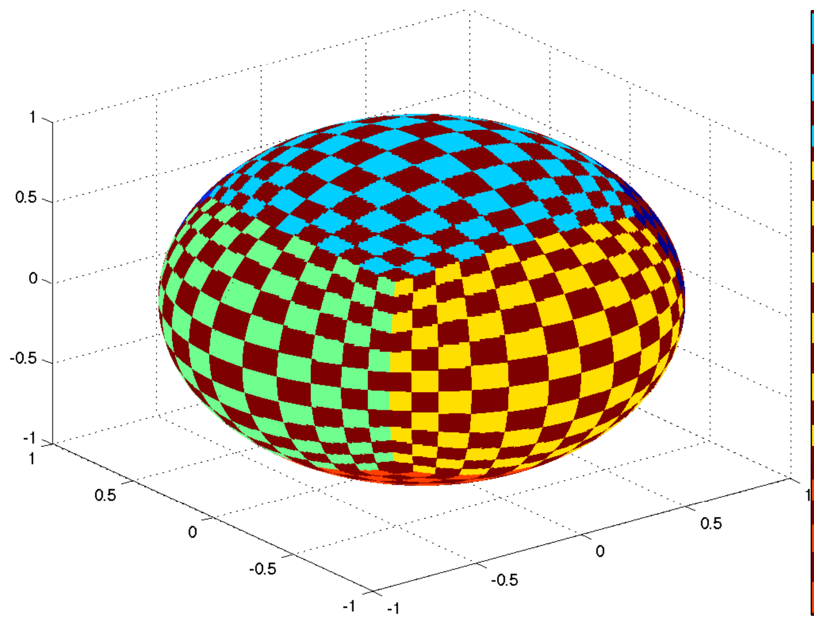
$\Rightarrow$ allows to find r given the two spectra

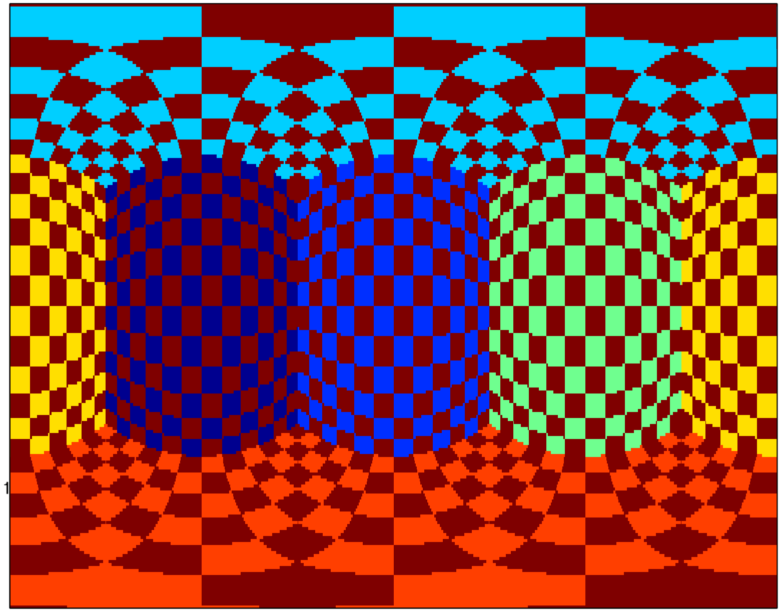
Representing functions on $\mathbb{S}^2$

- The Hough Spectrum is a function defined on the sphere:

$$\mathrm{HS} : \mathbb{S}^2 \to \mathbb{R}^+$$

- The trickiest part of the code is handling this representation.

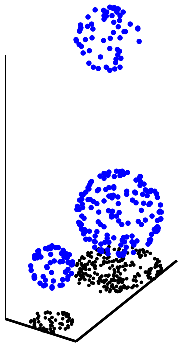
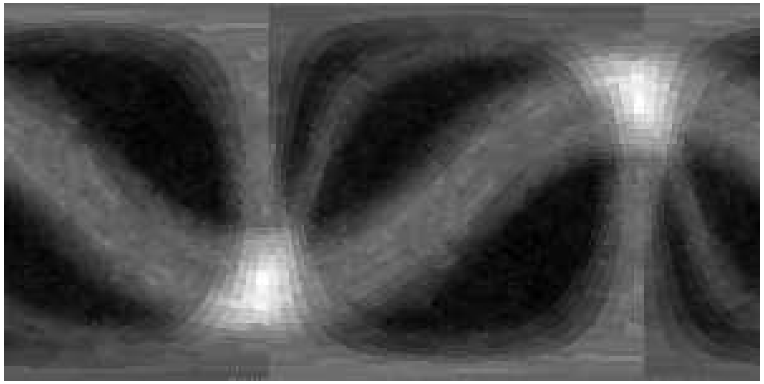


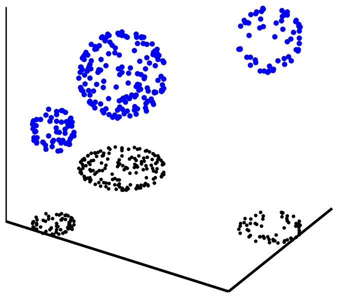
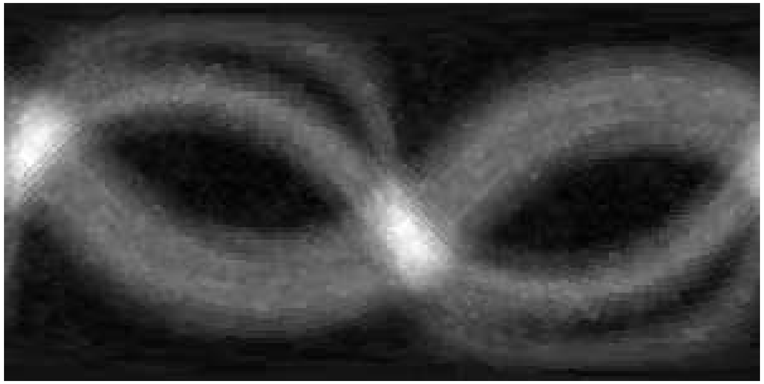


- In the next slides, Hough Spectra are shown projected to a cylinder.

Examples of spectra

- Hough Spectra of points clouds sampled from 3 spheres, two different positions.
- Hough Spectrum is the same, just rotated.

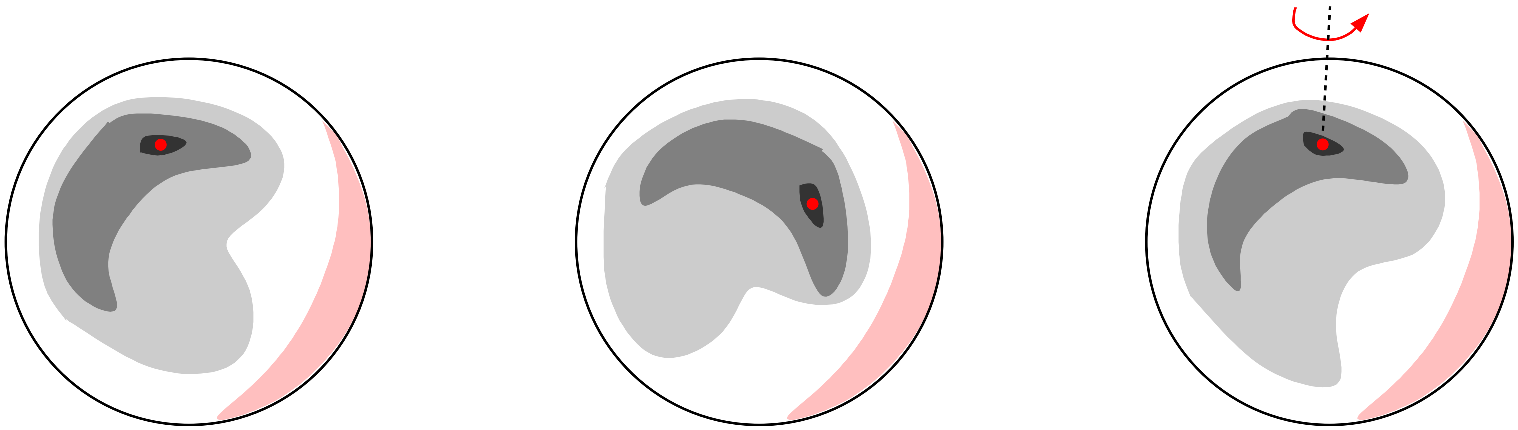
$i =$  $\mathrm{HS}(i) =$ 

$i \circ r =$  $\mathrm{HS}(i \circ r) =$ 

- Alignment is possible even though the surface histogram is flat
  - (Makadia's algorithm would fail)

Rotational matching problems

- **Problem**: let f_1, f_2 functions on the sphere, and $f_1 = f_2 \circ r$. Find r.
- The "elegant" way, Fourier Transform on SO(3) [Chirikjian], has complexity $O(nR^2) + O(R^3 \log^2 R)$.
- HSM3D uses a simple method to align two spherical images:
  - Align two candidate points (maxima)
  - Look for remaining rotation by cross correlation of the spheres.



1. first input — *2. second input* — *... partially aligned*

- Complexity is $O(nR^2) + O(R^2 logR)$, where n number of points, R angular resolution.

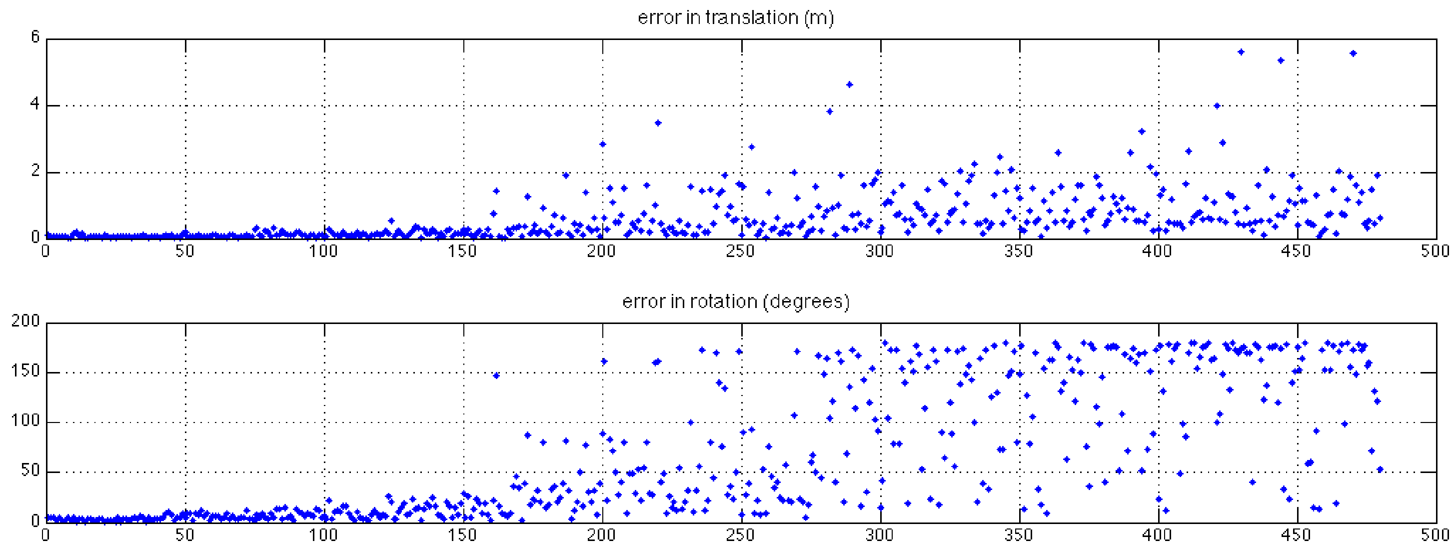
Results

Results for realignment after random rotation (data by Andreas Nuechter).

- ICP breaks down for rotation of >60 deg
- HSM3D has constant performance
    - cpu time: 3 s for 21,000 points, 3 deg resolution

translation errors

rotation errors



Initial angular error $\rightarrow$

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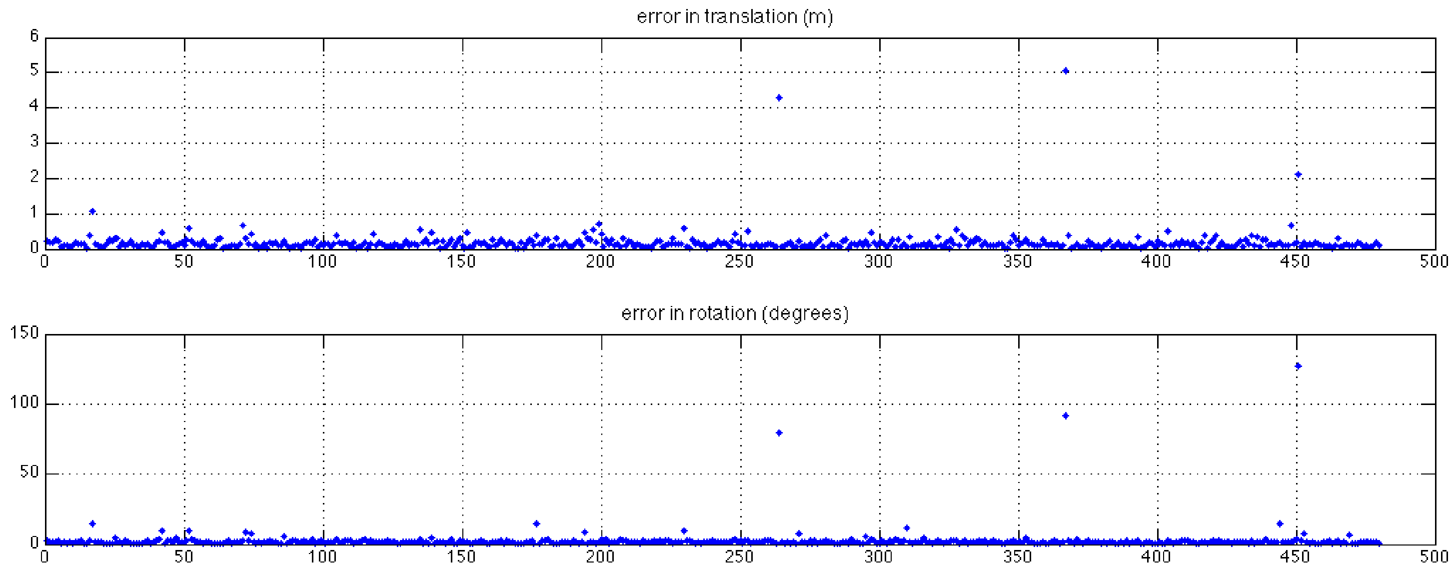

Initial angular error →

HSM3D Summary

Summary:

- HSM3D finds general 6DOF motions.
- HSM3D does not need planes/other features.
- HSM3D is global, and detects multiple hypotheses.
- HSM3D is resolution-complete.
- Matlab and C++ source code available (see URLs in the paper)

TODO:

- better engineering of the algorithm, tuning to range-finder data, faster implementations, ...
- ask Stefano Carpin for preliminary results on stereo vision data.