Joint inference and control: opportunities and challenges

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slides available at censi.mit.edu/slides
Perception is solved!
Perception is solved!

Wolfram Burgard
May 20

enjoys playing with google Project Tango — at Technische Fakultät.
As a robotics researcher, you shouldn’t compete with people doing “passive” perception.
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perception in robotics
perception **in** robotics

↓

perception **for** robotics
What is Robotics?
What is Robotics?

1. The business of adapting cool techniques in other fields to obtain a cute demo with a robot.
What is Robotics?

1. The business of adapting cool techniques in other fields to obtain a cute demo with a robot.

2. The scientific quest of understanding and replicating embodied intelligence.
What’s embodied intelligence about?
What’s embodied intelligence about?

- It’s (also) about doing well in the world using *limited resources*.
What’s embodied intelligence about?

- It’s (also) about doing well in the world using **limited resources**.

  *agent resources*

  - power
  - computation
  - memory
  - bandwidth
  - latency budget
What’s embodied intelligence about?

- It’s (also) about doing well in the world using **limited resources**.

- **designer resources** (“offline” resources)
  - design effort
  - prior knowledge

- **agent resources**
  - power
  - computation
  - memory
  - bandwidth
  - latency budget
complexity
complexity
complexity
complexity
complexity
Doing well with limited resources
Doing well with limited resources

- Here’s a task T; X watts of power; and Z bytes of memory. Design something that gives a reasonable answer in Y seconds.
Here’s a task $T$; $X$ watts of power; and $Z$ bytes of memory. Design something that gives a reasonable answer in $Y$ seconds.

$X = 500 \text{ watts}$

$Y = 100 \text{ milliseconds}$
Doing well with limited resources

- Here’s a task T; X watts of power; and Z bytes of memory. Design something that gives a reasonable answer in Y seconds.

\[ X = 50 \text{ milliwatts} \quad Y = 1 \text{ millisecond} \]

\[ X = 500 \text{ watts} \quad Y = 100 \text{ milliseconds} \]
Joint inference and control: opportunities and challenges
Joint inference and control: opportunities and challenges

solving the joint problem is more resource-efficient
The world/plant is a causal black box from $u$ to $y$. 

![Diagram showing the relationship between commands and observations in a world black box model.](image)
The world/plant is a causal black box from $u$ to $y$.

We need to design an agent/controller as a causal black box from $y$ to $u$. 

\[ u_k \in \mathcal{U} \quad \text{commands} \quad y_k \in \mathcal{Y} \quad \text{observations} \]

\[ y_k \in \mathcal{Y} \quad \text{observations} \quad u_k \in \mathcal{U} \quad \text{commands} \]
Markov assumption: $S$ are the states
Markov assumption: 
$S$ are the states
Markov assumption: $S$ are the states

- $s_0 \in S$
- $u_k \in \mathcal{U}$
- $y_k \in \mathcal{Y}$

**world**

$p(s_{k+1}|s_k, u_k)$

$p(y_k|s_k)$

**commands**

**motion model**

**observation model**


A (deterministic) agent is a tuple \( \langle \Gamma, f, g \rangle \)
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where \( \Gamma \) is any set representing the agent memory;
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\( f : \Gamma \times \mathcal{Y} \rightarrow \Gamma \) defines the memory dynamics;

\[
\gamma_{k+1} = f(\gamma_k, y_k)
\]
- A **(deterministic) agent** is a tuple \( \langle \Gamma, f, g \rangle \)
  where \( \Gamma \) is any set representing the agent memory;
  \( f : \Gamma \times Y \rightarrow \Gamma \) defines the memory dynamics;
  \( g : \Gamma \times Y \rightarrow U \) is the memory-to-command map.
The “canonical” probabilistic agent:
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\[ \Gamma = \text{beliefs (probability distributions on world’s state)} \]
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realistic

\[ \gamma_0 \in \Gamma \]

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\[ u_k = g(\gamma_k, y_k) \]

\[ \gamma_{k+1} = f(\gamma_k, y_k) \]

\[ \gamma_0 \in \Gamma \]

\[ y_k \in y \]

\[ u_k \in U \]

realistic

based on certainty-equivalence
The “canonical” probabilistic agent:

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Joint inference and control: opportunities and challenges

solving the joint problem is more resource-efficient
Joint inference and control: opportunities and challenges

- solving the joint problem is more resource-efficient
- There are many formalizations (only partially compatible)
Joint inference and control: opportunities and challenges

solving the joint problem is more resource-efficient

There are many formalizations (only partially compatible)

1. ...
2. ...
3. ...
4. ...
5. ...
Doing well with limited resources
Doing well with limited resources

1. Find an optimal agent that uses the fewest resources.
Doing well with limited resources

1. Find an optimal agent that uses the fewest resources.
2. Find a suboptimal agent with given resources bounds.
Offline design vs online execution
Offline design vs online execution

- problem
- spec
- synthesis
- algorithm
- agent
- description
Offline design vs online execution

\[ \text{problem} \quad \text{spec} \quad \text{synthesis} \quad \text{algorithm} \quad \text{agent} \quad \text{description} \]
Offline design vs online execution

- problem spec
- synthesis algorithm
- agent description

Instantiation:

- agent
- world
1. Minimality of sensing / control

\[ y_k \in y \quad \gamma_0 \in \Gamma \]

agent

\[ \gamma_{k+1} = f(\gamma_k, y_k) \]
\[ u_k = g(\gamma_k, y_k) \]

observations
commands

\[ u_k \in U \]
1. Minimality of sensing / control

- What can you do with minimal sensing / control?

\[ \gamma_0 \in \Gamma \]

\[ y_k \in y \]

\[ \gamma_{k+1} = f(\gamma_k, y_k) \]

\[ u_k = g(\gamma_k, y_k) \]

\[ u_k \in \mathcal{U} \]

observations

agent

commands
1. Minimality of sensing / control

- What can you do with minimal sensing / control?

O'Kane, LaValle. *On comparing the power of robots*. IJJR 2008

*Localization with limited sensing*. TRO 2007

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\gamma_0 &\in \Gamma \\
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- Sensing data is very redundant for place recognition

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Milford. *Vision-based place recognition: how low can you go?* IJRR 2013

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\gamma_0 & \in \Gamma \\
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2. Penalizing the cost of computation

Ortega, Braun. *Thermodynamics as a theory of decision-making with information-processing costs*, 2013

Braun, Ortega, Theodorou, Schaal. *Path Integral Control and Bounded Rationality*, 2011

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\[ pV - (1-p)V \]

updating information state takes work
*(physical work)*

variational problem

\[ \gamma_0 \in \Gamma \]
\[ y_k \in \mathcal{Y} \]
\[ u_k \in \mathcal{U} \]

\[ \gamma_{k+1} = f(\gamma_k, y_k) \]
\[ u_k = g(\gamma_k, y_k) \]
3. Penalizing the control information
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\[ y_k = s_k \]

\[ u_k \sim \pi_{s_k} \]

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Rubin, Shamir, Tishby. *Trading value and information in MDPs.* 2010

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= deviation from random policy =

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\[ = \text{deviation from random policy} = \mathbb{E}\left\{ \log \frac{\pi_s(u)}{\hat{\rho}(u)} \right\} \]

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4. Minimizing the agent-world bandwidth
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P1

P2
4. Minimizing the agent-world bandwidth

$P1$

$P2$
4. Minimizing the agent-world bandwidth

P1

P2
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\[ y_k = s_k \]

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agent

\[ u_k \in U \]
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“information to go” =

\[ y_k = s_k \]

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“information to go” = $D_{KL}(p_\pi \parallel \hat{p})$

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“information to go" = $D_{KL}(p_\pi \parallel \hat{p})$

\[ \text{distribution of states, actions under random policy} \]
4. Minimizing the agent-world bandwidth


"information to go" = \( D_{\text{KL}}(p_{\pi} \parallel \hat{p}) \)

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Reward

Bits flown between agent and world
4. Minimizing the agent-world bandwidth


"information to go" = $D_{KL}(p_\pi \mid\mid \hat{p})$

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reward

bits flown between agent and world
5. Minimality of representation (size of agent state)

- Most of the computation cost is in updating the representation.
- Penalize size of representation:

$$\min |\Gamma|$$
5. Minimality of representation (size of agent state)

1. Find an **optimal agent**
   that uses the **fewest resources**.

2. Find a **suboptimal agent**
   with given **resources bounds**.
5. Minimality of representation (size of agent state)

1. Find an **optimal agent**
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(a) A common belief

(b) An unlikely belief
5. Minimality of representation (size of agent state)

1. Find an **optimal agent** that uses the **fewest resources**.

2. Find a **suboptimal agent** with given **resources bounds**.
5. Minimality of representation


- A range-finder can be abstracted as a “gap sensor”

- Map can be represented as graphs

Q: Can we automatically synthesize minimal representations?
5. Minimality of representation

- **Task: find-object**
  - A robot must find a static object in a known environment.
5. Minimality of representation

- Task: find-object
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possible object positions
5. Minimality of representation

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- **Task: find-object**
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- **Sensors:**
  - Camera that detects object on sight.
  - Observable robot position (to be relaxed)
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- **Task: find-object**
  - A robot must find a static object in a known environment.

- **Sensors:**
  - Camera that detects object on sight.
  - Observable robot position (to be relaxed)

- **Actions:**
  - move (up, down, left, right)
  - declare where the intruder is
5. Minimality of representation

- Formalized as POMDP.
- Solution obtained from the MDP in belief space.
5. Minimality of representation

- Formalized as POMDP.
- Solution obtained from the MDP in belief space.
5. Minimality of representation

- Optimal agent only needs to represent optimally reachable beliefs.
5. Minimality of representation

- Optimal agent only needs to represent optimally reachable beliefs.

![Diagram showing belief space and reachable spaces](image)

Fig. 1. Belief space $\mathcal{B}$, reachable space $\mathcal{R}(b_0)$, and optimally reachable space $\mathcal{R}^*(b_0)$. Note that $\mathcal{R}^*(b_0) \subseteq \mathcal{R}(b_0) \subseteq \mathcal{B}$.

5. Minimality of representation

- “policy graph”: optimally reachable beliefs and corresponding optimal commands
5. Minimality of representation

- “policy graph”: optimally reachable beliefs and corresponding optimal commands

- Minimal representation is even smaller!

\[
\min |\Gamma|
\]
Q: What is the size of the **minimal representation**?
A: $|\Gamma| = 3$ states
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A: $|\Gamma| = 3$ states
Q: What is the size of the minimal representation?
A: $|\Gamma| = 3$ states

Here’s a minimal representation that we obtain automatically.
The size of the agent’s representation depends on the **sensorium power**.

more powerful \[\rightarrow\] less powerful
The size of the agent’s representation depends on the **sensorium power**.

Observable robot

position
The size of the agent’s representation depends on the **sensorium power**.

*more powerful*  

Observable robot position  

Use a range-finder for localization  

*less powerful*
The size of the agent’s representation depends on the **sensorium power.**

Use a range-finder for localization

Observable robot
position

horizon = 3

more powerful

less powerful
The size of the agent’s representation depends on the **sensorium power**.

Observable robot position

<table>
<thead>
<tr>
<th>more powerful</th>
<th>less powerful</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizon = 3</td>
<td>horizon = 2</td>
</tr>
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</table>

Use a range-finder for localization.
The size of the agent’s representation depends on the **sensorium power**.

- **more powerful**
- **less powerful**

**Observable robot position**

*Use a range-finder for localization*  

*horizon = 3*  

*horizon = 2*  

*horizon = 1*
The size of the agent’s representation depends on the **sensorium power**.

More powerful

<table>
<thead>
<tr>
<th>Observable robot position</th>
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<tr>
<td><strong>horizon = 3</strong></td>
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</tr>
<tr>
<td>3 states</td>
<td>5 states</td>
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<td>3 states</td>
<td>8 states</td>
</tr>
<tr>
<td>0 0</td>
<td>0 0 0 0</td>
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<tr>
<td>0 0</td>
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Less powerful
The size of the agent’s representation depends on the **sensorium power**.

**Observable robot position**

- **horizon = 3**
- **horizon = 2**
- **horizon = 1**

**Use a range-finder for localization**

- More powerful
- Less powerful
5. Minimality of representation

‣ “What is the simplest neural process that realizes the observed behavior?”
5. Minimality of representation

“What is the simplest neural process that realizes the observed behavior?”
Joint inference and control:
Joint inference and control: opportunities

solving the joint problem is more resource-efficient
Joint inference and control: opportunities

solving the joint problem is more resource-efficient

There are many formalizations (only partially compatible)
Joint inference and control: opportunities

solving the joint problem is more resource-efficient

There are many formalizations (only partially compatible)

1. Minimality of sensing / control
2. Penalizing computation
3. Penalizing control information
4. Penalizing agent-world bandwidth
5. Minimality of representation
Joint inference and control: \textbf{opportunities} and challenges

solving the joint problem is more resource-efficient

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Joint inference and control: opportunities and challenges

Death by generality

Q: What is robotics?

Q: What’s special about embodied intelligence?

solving the joint problem is more resource-efficient

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Joint inference and control: opportunities and **challenges**

- **Death by generality**
  - Q: What is robotics?
  - Q: What’s special about embodied intelligence?

- **Death by abstraction**
  - Q: What can we integrate within realistic architectures?

There are many formalizations (only partially compatible)

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